The basic model

- Generalization of previous models (formulated by L.Walras in 1874)
- The economy is composed of I consumers and J firms and L commodities (no distinction between inputs and outputs)
- There is a market system, prices are quoted for every commodity, and economic agents take these prices as independent of their individual actions
- Each consumer i is characterized by consumption set X_i with a well-behaved preferences; demand function is homogenneous of degree zero in prices
- Each firm j is characterized by a technology (or production set Y_j) that is nonempty and closed

Private ownership economy

- Private ownership economy – economy where consumer's wealth is derived from his ownership of endowments and from claims to profit shares firms.
- Firms are owned by consumers with initial endowment vector ω_i; each consumer has a non-negative ownership share $\theta_{ij} \geq 0$ in the profits of each firm ($\sum_i \theta_{ij} = 1$)
- An allocation is Pareto optimal if there is no waste (it is impossible to make any **consumer** better off without making some other consumer worse off).
- An allocation is feasible if $\sum_i x_{li} \leq \sum_i \omega_i + \sum_j y_{lj}$ for every l. The set of feasible allocations is nonempty, bounded, and closed.

Walrasian equilibrium

- An allocation (x^*, y^*) and a price vector $p=(p_1,\ldots, p_L)$ constitute a Walrasian (or competitive or market) equilibrium in a private ownership economy if:
- (1) for every j, $\boldsymbol{py_j} \leq \boldsymbol{py_j}^*$
- (2) for each comsumer i, x^* is the most preferred consumption in the budget set: $\boldsymbol{px_i} \leq \boldsymbol{p}\omega_{\boldsymbol{i}} + \sum_j \theta_{ij} \boldsymbol{py_j}^*$
- (3) $\sum_{\boldsymbol{i}} \boldsymbol{x}_i^* = \sum_i \omega_i + \sum_{\boldsymbol{j}} \boldsymbol{y}_j^*$

Equilibrium with transfers

- An allocation (x^*, y^*) and a price vector $p=(p_1,\ldots, p_L)$ constitute a price equilibrium with transfers if there is an assignment of wealth levels $(w_1, \ldots, w_I)$ with $\sum_i w_i = p\,\bar{\omega} + \sum_j p y_j^*$ such that:
- (1) for every j, $\boldsymbol{p}\boldsymbol{y}_j \leq \boldsymbol{p}\boldsymbol{y}_j^*$;
- (2) for each comsumer i, x_i^* is the most preferred consumption in the budget set: $\boldsymbol{p}\boldsymbol{x}_i \leq \boldsymbol{w}_i$;
- (3) $\sum_{\boldsymbol{i}} \boldsymbol{x}_i^* = \sum_i \omega_i + \sum_{\boldsymbol{j}} \boldsymbol{y}_j^*$

- A Walrasian equilibrium is a special case of a price equilibium with transfers.

First Fundamental Theorem of Welfare Economics

- If preferences are locally nonsatiated and (x^*, y^*, p) is a price equilibrium with transfers $\Rightarrow$ the allocation (x^*, y^*) is Pareto optimal.

$$\Downarrow$$

In particular, any Walrasian equilibrium allocation is Pareto optimal

Second Fundamental Theorem

- Requires more restrictions than 1-st theorem
- First we will weaken the concept of equilibrium (quasiequilibrium with transfers)
- Then give conditions at which the two coincide

Qasiequilibrium with transfers

- An allocation *(x^*, y^*)* and a price vector $p=(p_1,\ldots, p_L)$ constitute a <u>price quasiequilibrium with transfers</u> if there is an assignment of wealth levels $(w_1,\ldots,w_I)$ with $\sum_i w_i = p\,\bar{\omega} + \sum_j p y_j^*$ such that:
- (1) for every j, $\boldsymbol{p y_j \leq p y_j^*}$;
- (2) for each consumer i, if x_i is preferred to x_i^* , then : $\boldsymbol{p x_i \geq w_i}$;
- (3) $\sum_i x_i^* = \sum_i \omega_i + \sum_j y_j^*$

- Walrasian quasiequilibrium is a weaker notion than Walrasian equilibrium in that consumers are required to maximize preferences only relative to consumptions that cost strictly less than the available amount of wealth.

Second Fundamental Theorem

- If preferences are locally nonsatiated and convex $\Rightarrow$ for every Pareto optimal allocation (x^*, y^*) there is a price vector $(p_1,\ldots,p_L) \neq 0$ such that (x^*, y^*, p) is a quasiequilibrium with transfers.

$$\Downarrow$$

To identify Pareto optimal allocation by a planning authority, the **perfect information** (to compute the right supporting transfer levels) and **enforceability** (the power to enforce the necessary wealth transfers) are required.

- Any price equilibrium with transfers is a price quasiequilibrium with transfers.
- The converse is true under future conditions: any price quasiequilibrium with transfers is a price equilibrium with transfers, if consumption set is convex, preferences are continuous, wealth levels w_i are strictly positive.

Properties of Walrasian equilibria

- Walras' law: The value of the excess demand is zero for **any** price vector p, i.e.

$$\boldsymbol{p}\left[\sum_i \boldsymbol{x}_i\left(\boldsymbol{p}, \boldsymbol{p}\omega_i + \sum_j \theta_{ij}\pi_j(\boldsymbol{p})\right) - \sum_i \omega_i - \sum_j \boldsymbol{y}_j(\boldsymbol{p})\right] = \boldsymbol{p}\sum_i \boldsymbol{z}_i(\boldsymbol{p}) = \boldsymbol{p}\boldsymbol{z}(\boldsymbol{p}) = 0$$

- Market clearing: If demand equals supply in all markets but one and a price vector p is strictly positive, then demand must equal supply in all markets.
- Free goods: If some good a is in excess supply at a Walrasian equilibrium, it must be a free good, i.e. if p_a* is a Walrasian equilibrium and $z_a(p*) < 0$, then $p_a* = 0$.
- Desirability: If some price is zero, the aggregate excess demand for that good is strictly positive, i.e. if $p_l = 0$, then $z_l(p*) >> 0$.
- If all goods are desirable and $p*$ is a Walrasian equilibrium, then $z(p*) = 0$.